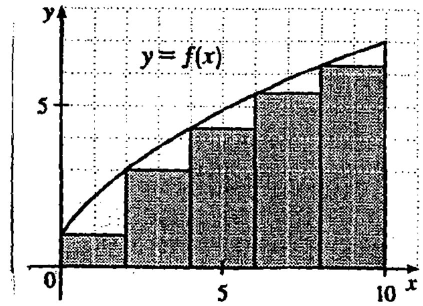
Chapter 5 Integrals

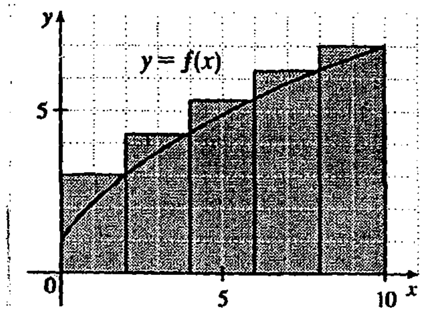
Section 5.1 Areas and Distances

1. (a) Since f is *increasing*, we can obtain a *lower* estimate by using *left* endpoints.



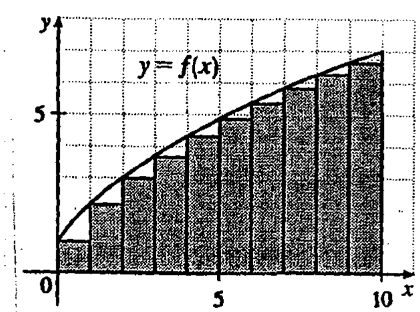
$$L_5 = \sum_{i=1}^{5} f(x_{i-1})\,\Delta x \quad \left[\Delta x = \tfrac{10-0}{5} = 2\right]$$
$$= f(x_0)\cdot 2 + f(x_1)\cdot 2 + f(x_2)\cdot 2 + f(x_3)\cdot 2 + f(x_4)\cdot 2$$
$$= 2\,[f(0) + f(2) + f(4) + f(6) + f(8)]$$
$$\approx 2\,(1 + 3 + 4.3 + 5.4 + 6.3) = 2\,(20) = 40$$

Since f is *increasing*, we can obtain an *upper* estimate by using *right* endpoints.

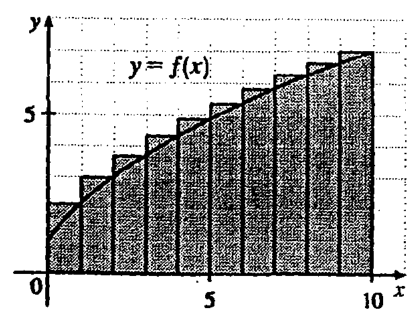


$$R_5 = \sum_{i=1}^{5} f(x_i)\,\Delta x$$
$$= 2\,[f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)]$$
$$= 2\,[f(2) + f(4) + f(6) + f(8) + f(10)]$$
$$\approx 2\,(3 + 4.3 + 5.4 + 6.3 + 7) = 2\,(26) = 52$$

(b) $$L_{10} = \sum_{i=1}^{10} f(x_{i-1})\,\Delta x \quad \left[\Delta x = \tfrac{10-0}{10} = 1\right]$$
$$= 1\,[f(x_0) + f(x_1) + \cdots + f(x_9)]$$
$$= f(0) + f(1) + \cdots + f(9)$$
$$\approx 1 + 2.1 + 3 + 3.7 + 4.3 + 4.9$$
$$+5.4 + 5.8 + 6.3 + 6.7 = 43.2$$



$$R_{10} = \sum_{i=1}^{10} f(x_i)\,\Delta x = f(1) + f(2) + \cdots + f(10)$$
$$= L_{10} + 1\cdot f(10) - 1\cdot f(0) \quad \left[\begin{array}{c}\text{add rightmost rectangle,}\\ \text{subtract leftmost}\end{array}\right]$$
$$= 43.2 + 7 - 1 = 49.2$$



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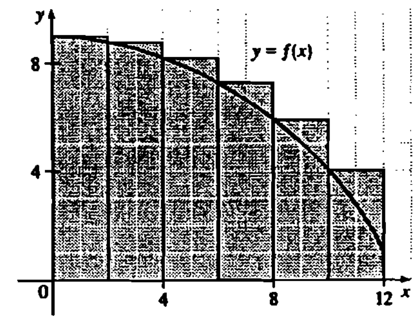
2. (a) (i) $L_6 = \sum_{i=1}^{6} f(x_{i-1})\,\Delta x \quad [\Delta x = \frac{12-0}{6} = 2]$

$= 2\,[f(x_0) + f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)]$

$= 2\,[f(0) + f(2) + f(4) + f(6) + f(8) + f(10)]$

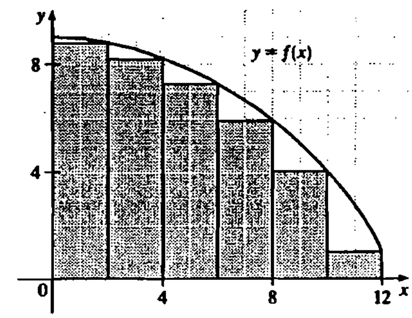
$\approx 2\,(9 + 8.8 + 8.2 + 7.3 + 5.9 + 4.1)$

$= 2\,(43.3) = 86.6$



(ii) $R_6 = L_6 + 2 \cdot f(12) - 2 \cdot f(0)$

$\approx 86.6 + 2\,(1) - 2\,(9) = 70.6$

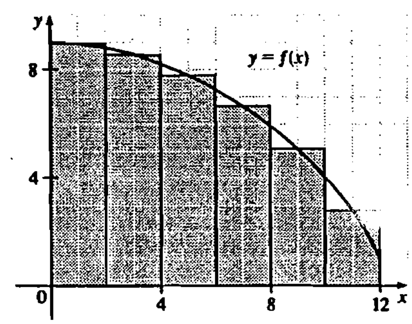


(iii) $M_6 = \sum_{i=1}^{6} f(x_i^*)\,\Delta x$

$= 2\,[f(1) + f(3) + f(5) + f(7) + f(9) + f(11)]$

$\approx 2\,(8.9 + 8.5 + 7.8 + 6.6 + 5.1 + 2.8)$

$= 2\,(39.7) = 79.4$



(b) Since f is *decreasing*, we obtain an *overestimate* by using *left* endpoints, that is, L_6.

(c) R_6 gives us an underestimate.

(d) M_6 gives the best estimate, since the area of each rectangle appears to be closer to the true area than the overestimates and underestimates in L_6 and R_6.

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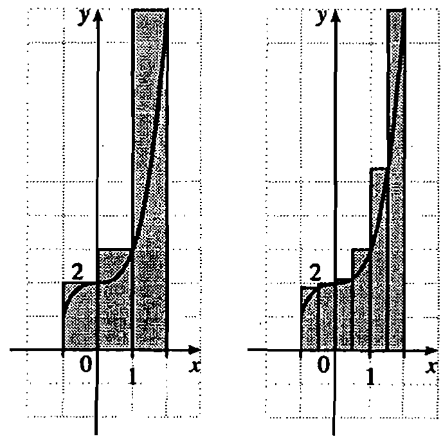
3. (a) $f(x) = x^3 + 2$ and $\Delta x = \frac{2-(-1)}{3} = 1 \Longrightarrow$

$R_3 = 1 \cdot f(0) + 1 \cdot f(1) + 1 \cdot f(2) = 1 \cdot 2 + 1 \cdot 3 + 1 \cdot 10 = 15.$

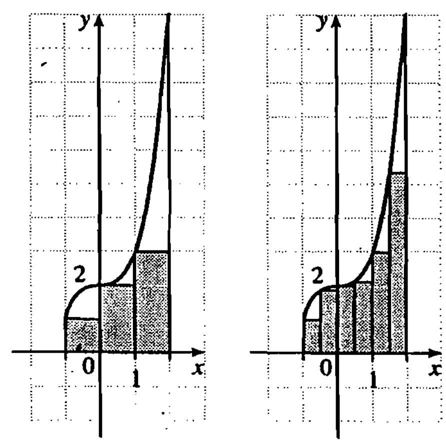
$\Delta x = \frac{2-(-1)}{6} = 0.5 \Longrightarrow$

$$R_6 = 0.5\left[f(-0.5) + f(0) + f(0.5) + f(1) + f(1.5) + f(2)\right]$$
$$= 0.5\,(1.875 + 2 + 2.125 + 3 + 5.375 + 10)$$
$$= 0.5\,(24.375) = 12.1875$$



(b) $L_3 = 1 \cdot f(-1) + 1 \cdot f(0) + 1 \cdot f(1) = 1 \cdot 1 + 1 \cdot 2 + 1 \cdot 3 = 6.$

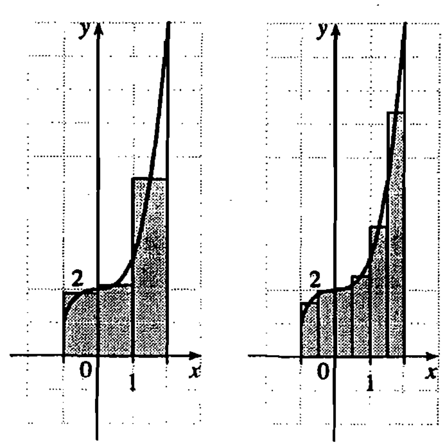
$$L_6 = 0.5\left[f(-1) + f(-0.5) + f(0) + f(0.5) + f(1) + f(1.5)\right]$$
$$= 0.5\,(1 + 1.875 + 2 + 2.125 + 3 + 5.375)$$
$$= 0.5\,(15.375) = 7.6875$$



(c) $M_3 = 1 \cdot f(-0.5) + 1 \cdot f(0.5) + 1 \cdot f(1.5)$

$= 1 \cdot 1.875 + 1 \cdot 2.125 + 1 \cdot 5.375 = 9.375.$

$$M_6 = 0.5\left[f(-0.75) + f(-0.25) + f(0.25) + f(0.75) + f(1.25) + f(1.75)\right]$$
$$= 0.5\,(1.578125 + 1.984375 + 2.015625 + 2.421875 + 3.953125 + 7.359375)$$
$$= 0.5\,(19.3125) = 9.65625$$



(d) M_6 appears to be the best estimate.

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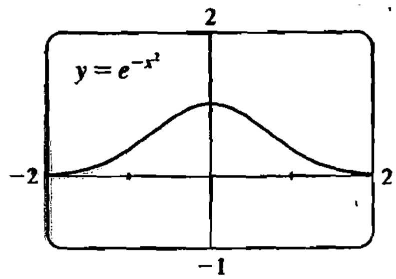
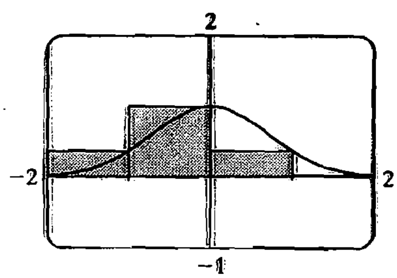
4. (a)

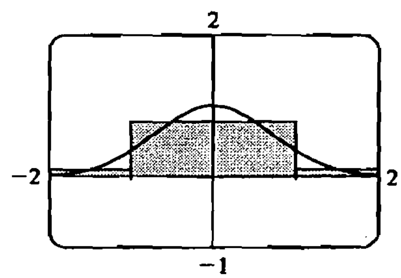


(b) $f(x) = e^{-x^2}$ and $\Delta x = \frac{2-(-2)}{4} = 1 \Longrightarrow$

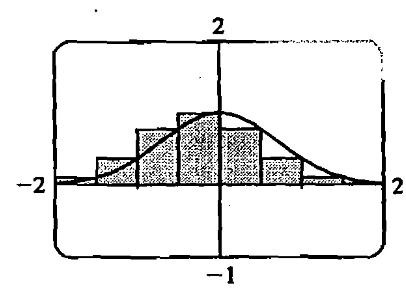
(i) $R_4 = 1 \cdot f(-1) + 1 \cdot f(0) + 1 \cdot f(1) + 1 \cdot f(2)$
$= e^{-1} + 1 + e^{-1} + e^{-4}$
≈ 1.754



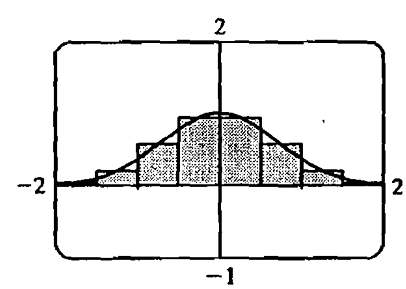
(ii) $M_4 = 1 \cdot f(-1.5) + 1 \cdot f(-0.5) + 1 \cdot f(0.5) + 1 \cdot f(1.5)$
$= e^{-2.25} + e^{-0.25} + e^{-0.25} + e^{-2.25}$
≈ 1.768



(c) (i) $R_8 = 0.5\,[f(-1.5) + f(-1) + f(-0.5) + f(0) + f(0.5) + f(1) + f(1.5) + f(2)]$
$= e^{-2.25} + e^{-1} + e^{-0.25} + 1 + e^{-0.25} + e^{-1} + e^{-2.25} + e^{-4}$
≈ 1.761



(ii) Due to the symmetry of the figure, we see that
$M_8 = (0.5)(2)\,[f(0.25) + f(0.75) + f(1.25) + f(1.75)]$
$= e^{-0.0625} + e^{-0.5625} + e^{-1.5625} + e^{3.0625}$
≈ 1.766



P360

Rectangle Approximation

(9) $L_6 = (0\,ft/s)(0.5\,s) + (6.2\,ft/s)(0.5\,s) + (10.8)(.5) +$
$(14.9)(.5) + (18.1)(.5) + (19.4)(.5)$
$= (.5)(69.4) = \boxed{34.7\,ft}$

$R_6 = 0.5(6.2 + 1.08 + 14.9 + 18.1 + 19.4 + 20.2)$
$= 0.5(89.6) = \boxed{44.8\,ft}$

(10) upper estimate by using the final velocity for each time interval

$d = \sum_{i=1}^{6} v(t_i)\Delta t_i = (185\,ft/s)(10\,s) + 319(5) + 447(5) +$
$742(12) + 1327(27) + 1445(3)$
$\boxed{54{,}694\,ft}$